

Optical Feshbach resonances through a molecular dark state: Efficient manipulation of p -wave resonances in fermionic ^{171}Yb atoms

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In a recent experiment by Yamazaki *et al.* [Phys.Rev. A **87** 010704 (R) (2013)], p -wave optical Feshbach resonance in fermionic ^{171}Yb atoms using purely long-range molecular excited states has been demonstrated. We theoretically show that, if two purely long range excited states of ^{171}Yb are coupled to the ground-state continuum of scattering states with two lasers, then it is possible to significantly suppress photoassociative atom loss by a dark resonance in the excited states. We present a general theoretical framework for creating a dark state in electronically excited molecular potential for the purpose of increasing the efficiency of an optical Feshbach resonance. This can be accomplished by properly adjusting the relative intensity, phase, polarizations and frequency detunings of two lasers. We present selective numerical results on atom loss spectra, p -wave elastic and inelastic scattering cross sections of ^{171}Yb atoms to illustrate the effects of the molecular dark state on optical Feshbach resonance.

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I. INTRODUCTION

Ability to control inter-particle interactions is important for exploring fundamental physics of many-particle systems in various interaction regimes. Towards this end, ultracold atomic gases offer unique opportunities since atom-atom interaction at low energy can be manipulated with external fields. In recent times, a magnetic Feshbach resonance (MFR) [1–3] has been extensively used to tune s -wave scattering length a_s of atoms over a wide range, facilitating the first demonstration of s -wave fermionic superfluidity in an atomic Fermi gas [4]. p -wave MFR has been experimentally observed in spin polarized ^{40}K [5] and ^6Li [6, 7] atomic gases, and theoretically analyzed [8, 9]. MFR has been used to produced p -wave Feshbach molecules [10, 11]. Atom-atom interaction can also be altered by an optical Feshbach resonance (OFR) proposed by Fedichev *et al.* [12]. Tunability of a_s by OFR has been experimentally demonstrated [13–15], albeit for a limited range. Recently, Yamazaki *et al.* [16] experimentally demonstrated p -wave OFR in fermionic ^{171}Yb atoms following an earlier theoretical proposal by Goel *et al.* [17].

In an OFR, a photoassociation (PA) laser is used to couple the scattering or free state of two S (ground) atoms to a bound state in an excited molecular potential asymptotically connecting to one ground (S) and another excited (P) atom. The loss of atoms due to spontaneous emission from the excited bound state is a severe hindrance to efficient manipulation of atom-atom interactions by an optical method. In the dispersive regime, the magnitude of the free-bound detuning is larger than both spontaneous and stimulated linewidths. In this regime, though the atom loss is mitigated, the change in elastic scattering amplitude is small. On the other hand, if the laser is tuned close to the free-bound transition frequency, there will be photoassociative formation of ex-

cited molecules which will eventually decay leading to drastic loss of atoms from the trap. Till now it is found that OFR is not an efficient method for tuning a_s as compared to MFR. It is therefore important to devise new methods to increase the efficiency of an OFR. Using an OFR one can manipulate not only s -wave but also higher partial wave interaction [18, 19] between ultracold atoms. Apart from this, development of an efficient OFR method is primarily necessary to manipulate two-body interactions in nonmagnetic atoms to which an MFR is not applicable. The question then arises: Is there any way out to suppress the loss of atoms in order to coherently and all optically manipulate atom-atom interactions? To address this, we carry out a theoretical study showing the manipulation of p -wave interaction in fermionic ^{171}Yb atoms by two lasers in different coupling regimes.

Here we show that it is possible to make an OFR significantly efficient by substantially suppressing atom loss by the method of dark state resonance [20, 21] in molecular excited states. Usually, a dark state resonance refers to the formation of a coherent superposition of two ground-state sub-levels of an atom or a molecule by two lasers. When a dark state is formed in ground-state sub-levels, an atom or a molecule can not effectively absorb a photon to reach to an excited state, and therefore no fluorescence light comes out. Dark resonance in ground-state sub-levels is well-known and plays an essential role in a number of coherent phenomena such as coherent population trapping (CPT) [22], laser cooling [23, 24] electromagnetically induced transparency (EIT) [21], stimulated Raman adiabatic passage (STIRAP) [25–27], slow light [28, 29] etc.. In contrast, dark resonances in atomic or molecular excited states have remained largely unexplored, because the excited states are in general too lossy. Now, with the accessibility of relatively long-lived excited states of alkaline earth-like atoms via intercombination PA transitions [30, 31], it is possible to create a

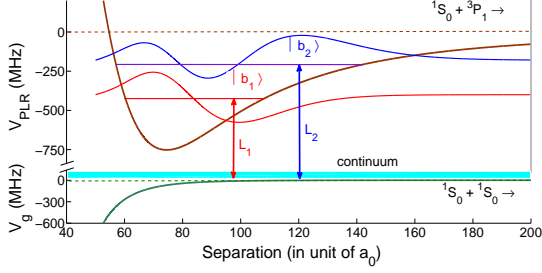


FIG. 1. (Color online) A schematic diagram for generating coherence between two PLR bound states $|b_1\rangle$ and $|b_2\rangle$ of $^{171}\text{Yb}_2$ by two OFR lasers L_1 and L_2 . The ground-state potential V_g (in unit of MHz) and PLR potential V_{PLR} (in unit of MHz) are plotted against the internuclear separation in unit of Bohr radius a_0 . Asymptotically, V_g corresponds to two separated atoms in electric $^1S_0 + ^1S_0$ states while V_{PLR} connect to two separated atoms in electronic $^1S_0 + ^3P_1$ states. $|b_1\rangle$ and $|b_2\rangle$ have vibrational quantum number $\nu_1 = 1$ and $\nu_2 = 2$, respectively; both have the same rotational quantum number T_e equal to either 1 or 3.

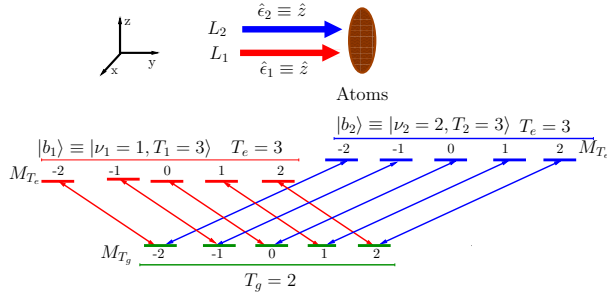


FIG. 2. (Color online) A schematic diagram showing how two lasers couple different molecular magnetic sub-levels of ground and excited states of ^{171}Yb atoms. Both lasers are linearly polarized along the z -axis. A pair of photons - one from L_1 (red) and the other from L_2 (blue) laser, couple a particular ground state magnetic sub-level M_{T_g} with two excited states having same angular momenta $T_e = 3$ and $M_{T_e} = M_{T_g}$, where M_{T_g} can take any of the 5 values from -2 to 2. Since the angular parts of the amplitudes for each such pair of transitions are the same, the magnitudes of the two transition amplitudes can be made equal by suitably adjusting the relative intensity of the two lasers. The phase of the two transition amplitudes is opposite due to the opposite vibrational parity of the two excited states.

coherent superposition or a coherence in ro-vibrational states of an excited molecule by two PA lasers. Though this laser-induced coherence has been discussed earlier in the contexts of d -wave OFR [19], vacuum-induced coherence [32] and rotational quantum beats [33], here we give an exposition of the crucial role it can play in manipulating an OFR. When two molecular ro-vibrational states belonging to the same electronically excited potential have the same rotational but different vibrational

quantum numbers, it is possible to generate a superposition of these two levels by applying two PA lasers. Under appropriate conditions, this superposed state can be protected against spontaneous emission leading to CPT in excited states [32].

Our purpose is to make use of molecular dark resonance to control OFR. For illustration, we investigate the manipulation of p -wave scattering properties of ^{171}Yb atoms with two OFR lasers L_1 and L_2 tuned to PA transitions to two purely long-range (PLR) bound states $|b_1\rangle$ and $|b_2\rangle$, respectively; as shown in Fig. 1. $|b_1\rangle$ and $|b_2\rangle$ have been chosen to have the same rotational quantum number $T_1 = T_2 = T_e$ but different vibrational quantum numbers $\nu_1 = 1$ and $\nu_2 = 2$, respectively. T may be chosen to be 1 or 3. By treating both lasers on an equal footing, we obtain results for any arbitrary optical-coupling regime. Our results show that the elastic scattering rate can exceed the inelastic rate by 5 orders of magnitude under the molecular dark resonance conditions in the strong-coupling regime. The atom loss can be almost completely eliminated by the use of the dark state. This leads to the huge enhancement in the efficiency of an all optical method for controlling atom-atom interactions.

The remainder of the paper is organized in the following way. In the next section we present our theory of two-laser OFR emphasizing on the essential idea behind the creation and utilization of molecular dark resonance for suppression of atom loss. In Sec.III, we apply this theoretical method to manipulate p -wave interactions of ^{171}Yb atoms. We then present numerical results and interpret them in Sec.IV. The paper is concluded in Sec.V.

II. THEORY

We first give the general idea behind molecular dark state-assisted OFR. We then specialize our theoretical method for p -wave OFR in fermionic ^{171}Yb atoms. A scheme of two-laser OFR in ^{171}Yb has been depicted in Fig. 1 which may also be used for a general scheme of two-laser OFR of ultracold atoms. Since photoassociative interaction between a nonzero partial-wave ($\ell \neq 0$) scattering state of two electronically ground-state atoms and an excited molecular bound state is essentially anisotropic, the usual theoretical method for s -wave OFR given by Bohn and Julienne [34] needs to be extended to include anisotropic effects. In our theoretical treatment, we take into account all the magnetic sub-levels of the ground and excited rotational levels. The two lasers are taken to be co-propagating and linearly polarized along the z -axis.

Two molecular bound states $|b_1\rangle$ and $|b_2\rangle$ supported by an excited-state potential are coupled to the continuum of unperturbed ground scattering states $|E'\rangle$ with collision energy E' by two PA lasers L_1 and L_2 . This leads to the formation of energy-normalized dressed state

[19].

$$|\psi_E\rangle = A_{1E} |b_1\rangle + A_{2E} |b_2\rangle + \int dE' C_{E'}(E) |E'\rangle \quad (2.1)$$

where A_{1E} , A_{2E} and $C_{E'}(E)$ are the dressed amplitudes given by

$$A_{1E} = D_{1E}^{-1} e^{i\phi_{L1}} [\Lambda_{1E} + \mathcal{L}_1^{(2)}] \quad (2.2)$$

$$A_{2E} = D_{2E}^{-1} e^{i\phi_{L2}} [\Lambda_{2E} + \mathcal{L}_2^{(1)}] \quad (2.3)$$

$$C_{E'}(E) = \delta(E - E') + \int dE'' \frac{A_{1E} \Lambda_{1E''} + A_{2E} \Lambda_{2E''}}{E - E''} \quad (2.4)$$

where ϕ_{L_n} is the phase of L_n laser. In the expression of A_{nE} , the first term $D_{nE}^{-1} \Lambda_{nE}$ is the amplitude that depends on direct free-bound coupling Λ_{nE} between the bound state $|b_n\rangle$ and the continuum due to laser L_n while the second term $D_{nE}^{-1} \mathcal{L}_n^{(n')}$ results from the cross-coupling between the bound states by the two lasers. Here the denominator $D_{nE} = E + \hbar\tilde{\delta}_n(E) + i\hbar G_n(E)$ with $\tilde{\delta}_n$ being the detuning of the L_n laser from the light-shifted n^{th} bound state,

$$\tilde{\delta}_n = \delta_n + \frac{1}{\hbar} (E_{b_n}^{\text{shift}} + E_{nn'}^{\text{shift}}) \quad (2.5)$$

where $\delta_n = \Delta_n + E_{b_n}/\hbar$ with $\Delta_n = \omega_{L_n} - \omega_A$ being the detuning of n^{th} laser from the frequency of atomic transition $S \rightarrow P$, E_{b_n} is the binding energy of $|b_n\rangle$ measured from the threshold of the excited-state potential and $E_{b_n}^{\text{shift}}$ is the light-shift of the bound state. Here $G_n(E) = \tilde{\Gamma}_n(E) + \Gamma_{nn'}(E)$ with $\tilde{\Gamma}_n(E)$ being the stimulated line width of n^{th} bound state due to L_n laser, $E_{nn'}^{\text{shift}}$ and $\Gamma_{nn'}$ are the terms that depends on the terms $\mathcal{L}_1^{(2)}$ and $\mathcal{L}_2^{(1)}$. Note that $\Gamma_{nn'}$ can be negative, but $G_n \geq 0$.

Equation (1) is derived without taking spontaneous emission into account. Following Bohn and Julienne [34], spontaneous decay can be included into the problem by introducing an ‘artificial’ open channel in the ground state manifold. Let the state of this artificial decay channel be denoted by $|E\rangle_{\text{art}}$ and its interaction with an excited state by V_{art} . The spontaneous emission linewidth can be identified with

$$\gamma_n = \frac{2\pi}{\hbar} |\langle E | V_{\text{art}} | b_n \rangle|^2 \quad (2.6)$$

As a result, G_n should be replaced by $G_n + \gamma_n$. The T -matrix element for inelastic process of transitions from the two correlated excited states to $|E\rangle_{\text{art}}$ is

$$\pi_{\text{art}} \langle E | V_{\text{art}} | \psi_E \rangle = \pi (\mathcal{V}_1 A_{1E} + \mathcal{V}_2 A_{2E}) \quad (2.7)$$

where $\mathcal{V}_n = \text{art} \langle E | V_{\text{art}} | b_n \rangle$. This gives the inelastic scattering cross section

$$\sigma_{\text{inel}} = \frac{4\pi g_s}{k^2} |\pi (\mathcal{V}_1 A_{1E} + \mathcal{V}_2 A_{2E})|^2 \quad (2.8)$$

where $g_s = 1(2)$ for two distinguishable(indistinguishable) atoms. The atom loss rate is given by $K_{\text{loss}} = \langle v_{\text{rel}} \sigma_{\text{inel}} \rangle$ where $\langle \dots \rangle$ implies thermal averaging over Maxwell-Boltzmann distribution of the relative velocity $v_{\text{rel}} = \hbar k / \mu$ with k being the relative wave number. Clearly, σ_{inel} or K_{loss} will vanish if $\mathcal{V}_1 A_{1E} = -\mathcal{V}_2 A_{2E}$, meaning

$$\frac{A_{1E}}{A_{2E}} = -\frac{\mathcal{V}_2}{\mathcal{V}_1} = -\sqrt{\frac{\gamma_2}{\gamma_1}} \quad (2.9)$$

is the condition for the onset of an excited molecular dark state that is protected against spontaneous emission. This condition can be fulfilled by suitably adjusting the relative intensity and phase between the two lasers, and the two detuning parameters Δ_1 and Δ_2 .

The elastic scattering amplitude can be obtained from asymptotic analysis of the dressed wavefunction $\psi_E(\mathbf{r}) = \langle \mathbf{r} | \psi_E \rangle$ where \mathbf{r} stands for the relative position vector of the two atoms. This can be conveniently done by partial-wave expansion as done in detail in Ref. [19]. Scattering properties of low-lying partial wave $\ell = 0$ (s -wave) or $\ell = 1$ (p -wave) or $\ell = 2$ (d -wave) can be optically manipulated by this two-laser OFR method. Which partial-wave will be most influenced by this method depends on the rotational quantum numbers of the two excited bound states and the temperature of the atomic cloud. While ultracold temperatures in the Wigner threshold law-regime are most suitable for manipulating s -wave collisions, temperatures slightly higher than Wigner threshold law-regime or temperatures near a shape resonance are appropriate for controlling higher partial-wave collisional properties. As an illustration we analyze manipulation of p -wave scattering properties of ^{171}Yb atoms with two-laser OFR in the next section.

III. TWO-LASER p -WAVE OFR IN ^{171}Yb ATOMS

For the bound states $|b_1\rangle$ and $|b_2\rangle$, we choose purely long range (PRL) molecular states of $^{171}\text{Yb}_2$. These states are fundamentally different from usual molecular bound states on several counts. First, these states are formed due to an interplay between resonant dipole-dipole and spin-orbit or hyperfine interactions in the excited atomic states. Second, their equilibrium position lies at a large separation well beyond the chemically interactive region of the overlap between the electron clouds of the two atoms. Third, the constituent atoms retain most of their atomic characters. Fourth, the potentials supporting such states are usually very shallow

allowing only a few vibrational levels to exist. Predicted about 35 years ago [35, 36], these states have been recently experimentally observed in alkali-metal [37–42], metastable helium [43] and fermionic ytterbium [44] atoms with PA spectroscopy. These states will be particularly useful for optical manipulation of p - or higher partial-wave atom-atom interactions. One of the major obstacles to higher partial-wave OFR stems from the fact that the partial-wave centrifugal barrier is too high for low-energy scattering wavefunction to be appreciable in the short-range region. With PLR states being used for OFR, PA transitions need not take place inside the barrier, opening up new scope for higher partial-wave OFR. The fact that the photoassociative atom loss mostly occurs at relatively short-range region makes PLR states a better choice for OFR in order to mitigate the loss.

In case of ^{171}Yb atoms, PLR bound states are formed due to an interplay between resonant dipole-dipole and hyperfine interactions. The PLR potential of $^{171}\text{Yb}_2$ of the state $^1S_0 + ^3P_1$ is obtained [44] by diagonalizing the adiabatic Hamiltonian

$$H_{\text{adia}} = \frac{\mathbf{d}_1 \cdot \mathbf{d}_2 - 3d_{1z}d_{2z}}{R^3} + a(\mathbf{i}_1 \cdot \mathbf{j}_1 + \mathbf{i}_2 \cdot \mathbf{j}_2) - \frac{C_6}{R^6} \quad (3.1)$$

where \mathbf{d}_n , d_{nz} , \mathbf{j}_n and \mathbf{i}_n denote the dipole moment of the atomic transition $^1S_0 \rightarrow ^3P_1$, z -component of the dipole moment, electronic and nuclear spin angular momentum, respectively, of the n^{th} ($=1,2$) atom. Here the parameters $a = 3957$ MHz, $C_6 = 2810$ a.u. and the magnitude of the transition dipole moment $d = 0.311$ a.u.. The axial projection Ω of the total electronic angular momentum $\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$ is not a good quantum number. The total nuclear spin angular momentum is $\mathbf{I} = \mathbf{i}_1 + \mathbf{i}_2$. The axial projection (Φ) of the total angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$ is a good quantum number. Further, when we include the rotation of the internuclear axis described by the partial-wave ℓ , the good quantum number are $\mathbf{T} = \mathbf{F} + \vec{\ell}$ and its projection M_T on the space-fixed axis. Now, effective excited potential is

$$V_e(r) = V_{\text{PLR}} + \frac{\hbar^2[T_e(T_e + 1) + \langle F^2 \rangle - 2\Phi^2]}{2\mu r^2} \quad (3.2)$$

where $V_{\text{PLR}}(r)$ is the PLR potential obtained by diagonalizing Eq.(3.1), μ is the reduced mass and T_e represents the rotational quantum number of the excited molecular state. The PLR potential that is accessible from p -wave ground-state scattering state via PA has depth of about 750 MHz and equilibrium separation at 75 Bohr radius [44]. The ground state potential is

$$V_g = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6} - \frac{C_8}{r^8} + \frac{\ell(\ell+1)}{2\mu r^2} \quad (3.3)$$

where $C_{12} = 1.03409 \times 10^9$ a.u., $C_8 = 1.93 \times 10^5$ a.u and $C_6 = 1931.7$ a.u [17, 45]. Ground-state ^{171}Yb has nuclear spin $i = \frac{1}{2}$ with no electronic spin. The ground-state collision in $s(p)$ partial-wave is characterized by the total nuclear spin $I = 0(1)$ due to the antisymmetry of total wave function. For molecular transitions, ground and excited states must have opposite parity. The selection rule is $\Delta T = 0, 1$ with $T = 0 \rightarrow T = 0$ being forbidden. For $\Phi_e \neq 0$ all T_e are allowed. For $\Phi_e = 0$ state only odd or even T_e are allowed [44, 46–48]. For p -wave we use 0^- PLR state, so only odd T_e are allowed.

Let us consider two PLR excited bound states of $^{171}\text{Yb}_2$ represented by $|b_\nu\rangle \equiv |T_e M_{T_e}\rangle_\nu$, where T_e is rotational quantum number, M_{T_e} denotes its projection along the z -axis of the laboratory frame and $\nu (= 1, 2)$ is the vibrational quantum number. As shown in Fig. 1, two OFR lasers L_1 and L_2 are used to induce photo-associative dipole coupling between the ground continuum state and the excited bound states with vibrational quantum numbers $\nu = 1$ and $\nu = 2$, respectively. Both the bound states have the same rotational quantum numbers either $T_e = 1$ or $T_e = 3$. Let the internal (rotational) state of the two ground-state atoms in molecular basis be denoted by $|T_g M_{T_g}\rangle$.

The dressed continuum of Eq. (2.1) can be derived following the method given in the appendix-A of Ref. [19]. It can be most conveniently done using the expansion in terms of molecular angular momentum basis functions or spherical harmonics. However, instead of $|\ell, m_\ell\rangle$ basis for the ground state, we use $|T_g, M_{T_g}\rangle$ basis for the present context.

The photoassociative loss of atoms is governed by the equation $\dot{n} = K_{\text{loss}} n^2$ where n is the atomic number density. Assuming a uniform number density \bar{n} the number of atoms remaining after the simultaneous action of both the lasers for a duration of τ is given by

$$N_{\text{remain}} = \frac{N_0}{1 + \bar{n} K_{\text{loss}} \tau} \quad (3.4)$$

where N_0 is the initial number of atoms.

Elastic scattering cross section

For the geometry and polarizations chosen for the two laser beams as schematically shown in Fig. 2, it is clear that the optical transitions couple ground and excited magnetic sub-levels $M_{T_1} = M_{T_2} = M_{T_g}$. We take $T_1 = T_2 = T_e$.

The asymptotic form of $\psi_E(\mathbf{r})$ is given by

$$\begin{aligned} \psi_E(\mathbf{r} \rightarrow \infty) \sim r^{-1} \sum_{M_{T_g}, M'_{T_g}} \psi_{T_g M_{T_g}}^{T_g M'_{T_g}}(r) Y_{T_g, M'_{T_g}}^*(\hat{k}) \\ \times \langle \hat{r} | T_g M_{T_g} \rangle \end{aligned} \quad (3.5)$$

where

$$\begin{aligned} \psi_{T_g M_{T_g}}^{T_g M_{T_g}'}(r) \langle \hat{r} | T_g M_{T_g} \rangle &= \sum_{\substack{m_l, m_I \\ m_l + m_I = M_T}} C_{lm_l Im_I}^{T_g M_{T_g}} \langle \hat{r} | lm_l Im_I, T_g M_{T_g} \rangle \left[\sin \left(kr - \frac{l\pi}{2} - \eta \right) - \pi F_{T_g M_{T_g}}^{T_g M_{T_g}'}(E, E') \right] \\ &\times \exp \left\{ i \left(kr - \frac{l\pi}{2} - \eta \right) \right\} \end{aligned} \quad (3.6)$$

where η is the background (in the absence of lasers) phase shift and $C_{lm_l Im_I}^{T_g M_{T_g}}$ is Clebsch-Gordan coefficient. Comparing this with Eq.(1) and using the expansion $C_{E'} \equiv \sum_{M_{T_g}, M_{T_g}'} C_{E', T_g M_{T_g}}^{T_g M_{T_g}'} Y_{T_g M_{T_g}}^*(\hat{k})$ we can relate $C_{E', T_g M_{T_g}}^{T_g M_{T_g}'} \langle r | E' \rangle_g \equiv \psi_{T_g M_{T_g}}^{T_g M_{T_g}'}(r)$. The \mathcal{T} matrix element is given by

$$\mathcal{T}_{TM_T, TM_T'} = -e^{i\eta} \sin \eta \delta_{m_l m_l'} \delta_{m_I m_I'} + \pi F_{T_g M_{T_g}}^{T_g M_{T_g}'} e^{2i\eta}. \quad (3.7)$$

Elastic scattering cross section is

$$\sigma_{el} = \frac{4\pi g_s}{k^2} \sum_{M_T, M_{T_g}'} |\mathcal{T}_{T_g M_{T_g}, T_g M_{T_g}'}|^2. \quad (3.8)$$

By using the expansion $A_{\nu E} = \sum_{T_g, M_{T_g}'} A_{\nu E}^{T_g M_{T_g}'}(M_{T_e}) Y_{T_g M_{T_g}}^*(\hat{k})$ we have [19]

$$F_{T_g M_{T_g}}^{T_g M_{T_g}'} = \sum_{\nu, M_{T_e}} A_{\nu E}^{T_g M_{T_g}'} \Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E). \quad (3.9)$$

Here $\Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E)$ is the amplitude for optical transition $|T_e, M_{T_e}\rangle_\nu \rightarrow |E, T_g M_{T_g}\rangle$ due to L_ν laser, where $|E, T_g M_{T_g}\rangle$ represents unperturbed (laser-free) scattering state for (T_g, M_{T_g}) quantum numbers. Explicitly,

$$A_{\nu E}^{T_g M_{T_g}'}(M_{T_e}) = D_\nu^{-1} \left[\left\{ \Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E) \right\}^* + \mathcal{L}_\nu^{\nu'} \right] \quad \nu' \neq \nu \quad (3.10)$$

where $D_\nu = \zeta_\nu + i\mathcal{J}_\nu/2$ with $\mathcal{J}_\nu = \Gamma_\nu + \Gamma_{\nu\nu'} + \gamma_\nu$, $\zeta_\nu = E + \hbar\Delta_\nu - (E_\nu + E_\nu^{\text{shift}} + E_{\nu\nu'}^{\text{shift}})$. Here

$$\mathcal{L}_\nu^{\nu'} = \xi_{\nu'}^{-1} \mathcal{K}_{\nu\nu'}. \quad (3.11)$$

with $\xi_\nu(E) = E + \hbar\Delta_\nu - (E_\nu + E_\nu^{\text{shift}}) + i\Gamma_\nu/2$ and

$$\mathcal{K}_{\nu\nu'} = \left(\mathcal{V}_{\nu\nu'} - i\frac{1}{2}\hbar\mathcal{G}_{\nu\nu'} \right) \quad (3.12)$$

is the cross coupling between two excited bound states

induced by the two lasers, where

$$\mathcal{V}_{\nu\nu'} = \sum_{TM_T} \mathcal{P} \int \frac{\Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E') \left\{ \Lambda_{T_g M_{T_g}}^{\nu', T_e M_{T_e}}(E') \right\}^*}{E - E'} dE' \quad (3.13)$$

$$\mathcal{G}_{\nu\nu'} = \frac{2\pi}{\hbar} \sum_{TM_T} \Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E) \left\{ \Lambda_{T_g M_{T_g}}^{\nu', T_e M_{T_e}}(E) \right\}^*. \quad (3.14)$$

The stimulated line width of ν th bound state due to the ν th laser only is

$$\Gamma_\nu(E) = \frac{2\pi}{\hbar} \sum_{M_{T_g}} |\Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E)|^2 \quad (3.15)$$

and the corresponding light shift is

$$E_\nu^{\text{shift}} = \sum_{M_T} \mathcal{P} \int \frac{\Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E') \left\{ \Lambda_{T_g M_{T_g}}^{\nu, T_e M_{T_e}}(E') \right\}^*}{E - E'} dE'. \quad (3.16)$$

The terms that arise due to cross coupling $\mathcal{K}_{\nu\nu'}$ are

$$E_{\nu\nu'}^{\text{shift}} = \text{Re}[\xi_{\nu'}^{-1} \mathcal{K}_{\nu\nu'} \mathcal{K}_{\nu'\nu}] \quad \text{and} \quad (3.17)$$

$$\Gamma_{\nu\nu'} = -2\text{Im}[\xi_{\nu'}^{-1} \mathcal{K}_{\nu\nu'} \mathcal{K}_{\nu'\nu}]. \quad (3.18)$$

IV. RESULTS AND DISCUSSIONS

For numerical illustration, we consider that the two OFR lasers L_1 and L_2 couple $T_g = 2$ to two PLR vibrational states $\nu = 1$ (b_1) and $\nu = 2$ (b_2), respectively, with same $T_e = 3$. As discussed in the previous section, from symmetry considerations, the PLR states chosen will be accessible only from odd partial-waves (odd ℓ) and nuclear spin triplet ($I = 1$). As per the selection rules $\Delta T = 0, 1$ the excited rotational state $T_e = 3$ will be accessible from the ground $T_g = 2$ or $T_g = 4$, which means from $\ell = 1$ or $\ell = 3$. We assume that the contributions from $\ell = 3$ is negligible due to low temperature. Further, we select laser detunings and intensities such that the optical couplings to the levels $T_1 = T_2 = 1$ are negligible. We can thus restrict our study to $T_g = 2$ only. These two states have the binding energies $E_{b_1} = -355.4$ MHz

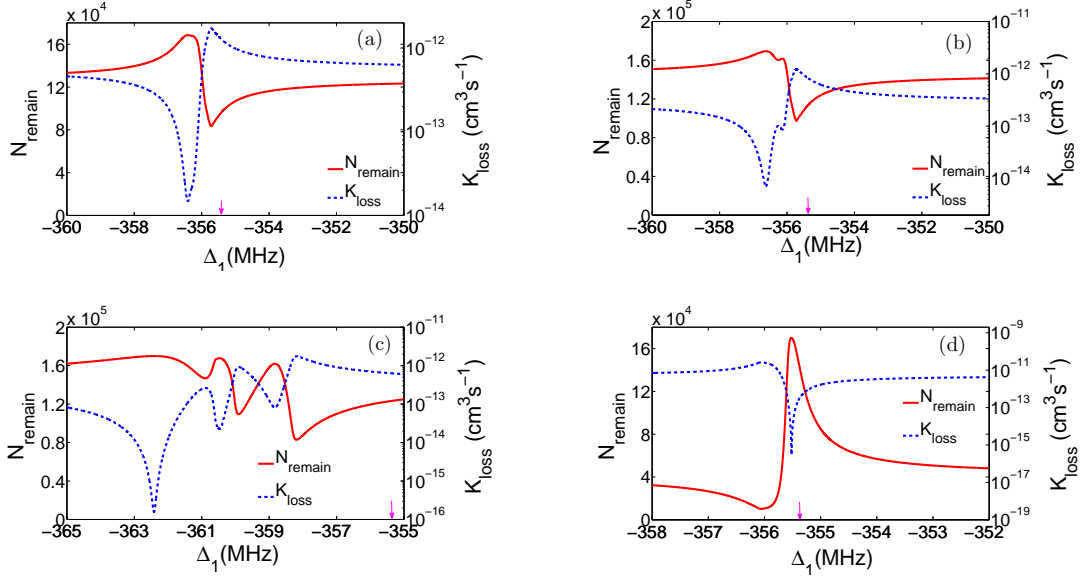


FIG. 3. (Color online) Number of atoms N_{remain} (solid) remaining in the trap after the two OFR lasers have acted for the time duration of 30 ms and PA rate K_{loss} (dashed) in unit of cm^3s^{-1} are plotted against the detuning Δ_1 (in MHz) of the first laser from the atomic transition frequency at temperature $T = 8 \mu\text{K}$ for different laser intensities: (a) $I_1 = 1\text{Wcm}^{-2}$ ($\Gamma_1 = 212.9$ KHz) and $I_2 = 10\text{Wcm}^{-2}$ ($\Gamma_2 = 4.3$ MHz), (b) $I_1 = 1\text{Wcm}^{-2}$ ($\Gamma_1 = 212.9\text{KHz}$) and $I_2 = 20\text{Wcm}^{-2}$ ($\Gamma_2 = 8.6$ MHz), (c) $I_1 = 10\text{Wcm}^{-2}$ ($\Gamma_1 = 2.1$ MHz) and $I_2 = 20\text{Wcm}^{-2}$ ($\Gamma_2 = 8.6$ MHz), (d) $I_1 = 0.2\text{Wcm}^{-2}$ ($\Gamma_1 = 106$ KHz) and $I_2 = 0.1\text{Wcm}^{-2}$ ($\Gamma_2 = 43$ KHz). The other parameters are: $\delta_2 = 0$ and laser phase difference $\theta = 0$. The arrow indicates the binding energy $E_{b_1} = -355.4$ MHz of the unperturbed bound state $|b_1\rangle$.

$E_{b_2} = -212.4$ MHz measured from the threshold of excited potential. The laser irradiation time is taken to be 30 ms, and the average atomic density $\bar{n} = 2 \times 10^{13} \text{cm}^{-3}$ [16, 49], temperature is $8 \mu\text{K}$ and initial atom number $N_0 = 1.7 \times 10^5$. Spontaneous line widths of the bound states are $\gamma_1 = \gamma_2 = 2\pi \times 364$ KHz [16, 49].

Fig. 3(a)-(d) displays the number of atoms remaining N_{remain} and PA loss rate K_{loss} as a function of detuning of the first laser $\Delta_1 = \omega_{L_1} - \omega_A$ from the atomic transition frequency ω_A (asymptote of the PLR potential) for different intensities of the two lasers. In Figs. 3(a) and 3(b), the first laser intensity is fixed in the intermediate coupling regime ($\Gamma_1 \leq \gamma$) while the intensity of the second laser is set for the strong-coupling regime ($\Gamma_2 > \gamma$). For Fig. 3(c), the intensities of both lasers are in the strong-coupling regimes ($\Gamma_1 \gg \gamma, \Gamma_2 \gg \gamma$). K_{loss} (dashed curves) in Figs. 3(a)-(d) exhibit a prominent minimum, the values of K_{loss} at the minimum being about $3.8 \times 10^{-14} \text{cm}^3\text{s}^{-1}$, $7.0 \times 10^{-15} \text{cm}^3\text{s}^{-1}$ and $1.5 \times 10^{-16} \text{cm}^3\text{s}^{-1}$, $2.96 \times 10^{-16} \text{cm}^3\text{s}^{-1}$, respectively. Let $\Delta_{1\text{min}}$ be the value of Δ_1 at which the minimum occurs. N_{remain} as a function of Δ_1 exhibits complementary behavior to that of K_{loss} , attaining the maximum exactly at $\Delta_{1\text{min}}$. The value N_{remain} at $\Delta_{1\text{min}}$ is nearly equal to N_0 implying that the loss of atoms is negligible at $\Delta_{1\text{min}}$. This occurs due to the formation of molecular dark state leading to the destructive quantum interference

between spontaneous emission transition pathways. The fact that the spectra for both K_{loss} and N_{remain} in Fig.3 are asymmetric with a prominent minimum and a maximum is indicative of the occurrence of quantum interference. In case of one-laser OFR in the weak coupling regime ($\Gamma \ll \gamma$), the spectra have symmetric Lorentzian shape. We notice that $\Delta_{1\text{min}}$ exhibits shifts towards the lower values of ω_{L_1} as we move from Fig. 3(a) to Fig. 3(c). The increasing red shifts due to increasing laser intensity I_1 are consistent with the calculated light shifts. Light shift $E_{\nu_n, M_{T_n}}^{\text{shift}}$ of $|b_n\rangle$ due to L_n laser only is proportional to the laser intensity I_n , and does not depend on the detunings. In contrast $E_{\nu\nu'}^{\text{shift}}$ depends on both laser intensities and the detuning $\Delta\nu'$. We have found $E_{\nu_1, M_{T_1}=0}^{\text{shift}}/I_1 = -1.62 \text{ MHzW}^{-1}\text{cm}^2$, $E_{\nu_1, M_{T_1}=1}^{\text{shift}}/I_1 = -1.44 \text{ MHzW}^{-1}\text{cm}^2$, $E_{\nu_1, M_{T_1}=2}^{\text{shift}}/I_1 = -0.904 \text{ MHzW}^{-1}\text{cm}^2$ and $E_{\nu_1, M_{T_2}=0}^{\text{shift}}/I_2 = -1.82 \text{ MHzW}^{-1}\text{cm}^2$, $E_{\nu_1, M_{T_2}=1}^{\text{shift}}/I_2 = -1.61 \text{ MHzW}^{-1}\text{cm}^2$, $E_{\nu_1, M_{T_2}=2}^{\text{shift}}/I_2 = -1.00 \text{ MHzW}^{-1}\text{cm}^2$. The calculated values of E_{12}^{shift} and E_{21}^{shift} are given in the table I and II, respectively.

From Fig. 3(a)-(c), we further notice that the width of the dip in K_{loss} or equivalently the width of the maximum in N_{remain} increases as we move from Fig. 3(a) to Fig. 3(c). This means that the strong-coupling regimes

TABLE I. The values of $E_{12}^{\text{shift}}(M_T)$ in MHz are shown for the laser intensities as used in Fig. 3 with $\delta_2 = 0$.

I_1 W/cm ²	I_2 W/cm ²	E_{12}^{shift} ($M_T = 0$)	E_{12}^{shift} ($M_T = 1$)	E_{12}^{shift} ($M_T = 2$)
1	10	1.09	0.975	0.60
1	20	1.10	0.980	0.610
10	20	11.03	9.8	6.10
0.2	0.1	0.117	0.098	0.047

TABLE II. The values of $E_{21}^{\text{shift}}(M_T)$ in MHz are shown for $\Delta_1 = \Delta_{1\min}$.

I_1 W/cm ²	I_2 W/cm ²	E_{21}^{shift} ($M_T = 0$)	E_{21}^{shift} ($M_T = 1$)	E_{21}^{shift} ($M_T = 2$)
1	10	12.58	38.56	-47.7
1	20	25.66	82.43	-77.72
10	20	43.49	41.52	55.90
0.2	0.1	0.102	0.089	0.050

with a molecular dark resonance are robust for efficient manipulation of atom-atom interactions. The oscillations in Fig. 3 result from the laser-induced coherence between the two excited bound states described by the term $\mathcal{L}_{\nu\nu'}$. This laser induced coherence is important in the strong-coupling regimes because it comes into play due to nonlinear effects. One photon from L_1 laser excites the bound state $|b_1\rangle$ which then emits another photon by stimulated emission. When this absorption-emission cycle is followed by the excitation of the other bound state $|b_2\rangle$ by one photon from L_2 laser, we have the coherence $\mathcal{L}_{\nu\nu'=1}$. The maximum in N_{remain} vs. Δ_1 and the spec-

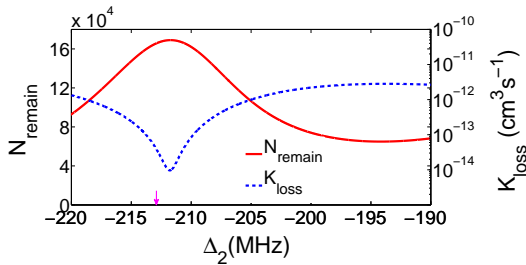


FIG. 4. (Color online) Same as in FIG. 3(b) but as a function of Δ_2 at $\Delta_1 = \Delta_{1\min}$.

tral asymmetry also appear in weak-coupling regimes for both lasers as shown in Fig. 3(d). At collision energy $E = 8 \mu\text{K}$ the square of the Franck-Condon overlap integral for the transition to $|b_2\rangle$ is about 2 times larger than that to $|b_1\rangle$. Therefore, if we fix the intensity of L_1 laser at half the intensity of L_2 laser, we expect that the two

free-bound transition amplitudes with $M_{T_g} = M_{T_e}$ will be almost same. Then, in the weak-coupling regimes, we would expect the maximum in N_{remain} to occur at or near the energy of the unperturbed bound state $|b_1\rangle$ if the maximum arises due to the dark state. In fact, the solid curve in Fig. 3(d) has the maximum near the binding energy of $|b_1\rangle$.

Keeping the value of Δ_1 fixed at $\Delta_{1\min}$ which happens to be $\tilde{\delta}_1 \simeq 0$, we plot K_{loss} and N_{remain} as a function of Δ_2 in Fig. 4. for the parameters as in Fig. 3(b). This shows that K_{loss} has a broad minimum when Δ_2 is tuned near the resonance to $|b_2\rangle$. The loss of atoms is almost nil for the parameters at which the minimum in K_{loss} occurs.

We next show elastic scattering cross section σ_{el} as a function of Δ_1 and compare it with the inelastic one σ_{inel} in Fig. 5. keeping other parameters fixed as in Fig. 3(b). σ_{el} is 6 orders of magnitude larger at and near $\Delta_{1\min}$ where the minimum of σ_{inel} occurs due to the dark state. Note that the back-ground (in the absence of OFR) elastic cross section is negligible ($\sim 10^{-19} \text{ cm}^2$). We have found that σ_{el} and σ_{inel} are of comparable magnitudes in the weak-coupling regimes.

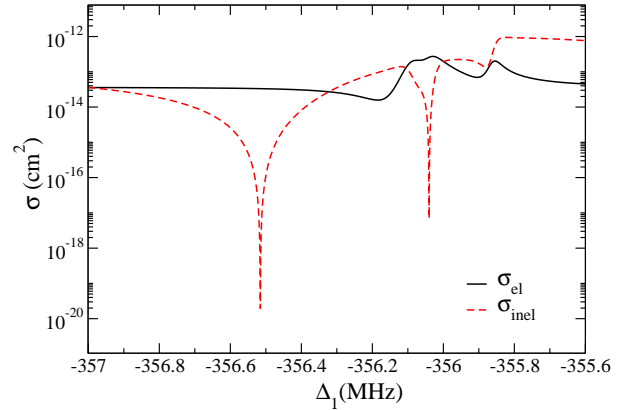


FIG. 5. (Color online) σ_{el} and σ_{inel} are plotted as a function Δ_1 for collision energy $E = 8 \mu\text{K}$. All other parameters are same as in Fig. 3(b).

V. CONCLUSION

In conclusion, we have demonstrated highly efficient manipulation of p -wave interactions between fermionic ^{171}Yb atoms with a new optical method using two lasers in the strong-coupling regime. This method relies on creating a molecular dark state in the electronically excited potential, leading to the inhibition of photoassociative atom loss. It is possible when two excited molecular bound states coupled to the continuum of scattering states by the two lasers have the same rotational quantum

number; and all pairs of excited sub-levels having the same magnetic quantum number are coupled a ground-state sub-level by the two lasers as schematically depicted in Fig. 2. This ensures the cancellations of spontaneous emissions from all the excited sub-levels when the conditions for the formation of the dark state are fulfilled. The efficiency of our method may be characterized by a number of parameters: (1) the ratio N_{remain}/N_0 of the number of atoms remaining N_{remain} to the initial number N_0 , (2) the ratio $\sigma_{\text{el}}/\sigma_{\text{inel}}$ of the elastic to inelastic scattering cross sections and (3) the ratio of the width of the dip in atom loss rate to the spontaneous line width. When the conditions for dark state are satisfied in the strong-coupling regimes for both the lasers, we have the results $N_{\text{remain}}/N_0 \simeq 1$, $\sigma_{\text{el}}/\sigma_{\text{inel}} \sim 10^4 - 10^6$ and the

ratio between the two widths can be much larger than 1. Considering the velocity of the atoms to be a few cm s^{-1} , the number density $\sim 10^{13} \text{ cm}^{-3}$ we find elastic rate $\sim 1 \text{ s}^{-1}$ while the inelastic rate $\sim 10^{-5} \text{ s}^{-1}$. These numbers indicate that it is possible to manipulate atom-atom interactions efficiently by the optical method presented in this paper.

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